

**QF605 Fixed Income (AY2021/2022)**

Prepared By:

**Dani Surya Pangestu**

**Gabriel Woon​**

**Gabriel Tan**

**Kenneth Chong**

**Peter Chettiar**

**Wong Yong Wen**

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# Part I – Bootstrapping

## OIS Discount Factor

Equations Used (Example for Year 2):

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Using the above equations, linear interpolation for timeframes without corresponding overnight interest swaps (OIS) and Brent’s method as our root finder to solve for fi, we derived the OIS discount factor table as shown below:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Tenor** | **OIS DF** | **Tenor** | **OIS DF** | **Tenor** | **OIS DF** | **Tenor** | **OIS DF** | **Tenor** | **OIS DF** | **Tenor** | **OIS DF** |
| 0.5 | 0.99875 | 5.5 | 0.97973 | 10.5 | 0.95311 | 15.5 | 0.92483 | 20.5 | 0.89737 | 25.5 | 0.87073 |
| 1 | 0.99701 | 6 | 0.97728 | 11 | 0.95024 | 16 | 0.92204 | 21 | 0.89467 | 26 | 0.86811 |
| 1.5 | 0.99527 | 6.5 | 0.97484 | 11.5 | 0.94738 | 16.5 | 0.91927 | 21.5 | 0.89198 | 26.5 | 0.86550 |
| 2 | 0.99353 | 7 | 0.97241 | 12 | 0.94453 | 17 | 0.91650 | 22 | 0.88929 | 27 | 0.86289 |
| 2.5 | 0.99177 | 7.5 | 0.96965 | 12.5 | 0.94169 | 17.5 | 0.91375 | 22.5 | 0.88662 | 27.5 | 0.86030 |
| 3 | 0.99002 | 8 | 0.96690 | 13 | 0.93886 | 18 | 0.91099 | 23 | 0.88395 | 28 | 0.85770 |
| 3.5 | 0.98807 | 8.5 | 0.96416 | 13.5 | 0.93604 | 18.5 | 0.90826 | 23.5 | 0.88129 | 28.5 | 0.85513 |
| 4 | 0.98612 | 9 | 0.96142 | 14 | 0.93322 | 19 | 0.90552 | 24 | 0.87864 | 29 | 0.85255 |
| 4.5 | 0.98415 | 9.5 | 0.95870 | 14.5 | 0.93042 | 19.5 | 0.90280 | 24.5 | 0.87600 | 29.5 | 0.84999 |
| 5 | 0.98218 | 10 | 0.95598 | 15 | 0.92761 | 20 | 0.90008 | 25 | 0.87336 | 30 | 0.84742 |

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## LIBOR Discount Factor

Equations Used (Example for Year 2):

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Using the above equations, linear interpolation for timeframes without corresponding Interest Rate Swaps (IRS) and Brent’s method as our root finder to solve for L(T­i, Ti+1), we derived the Libor discount factor table as shown below:

|  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| **Tenor** | **Libor DF** | **Tenor** | **Libor DF** | **Tenor** | **Libor DF** | **Tenor** | **Libor DF** | **Tenor** | **Libor DF** | **Tenor** | **Libor DF** |
| 0.5 | 0.98765 | 5.5 | 0.83280 | 10.5 | 0.67855 | 15.5 | 0.53679 | 20.5 | 0.39899 | 25.5 | 0.30671 |
| 1 | 0.97258 | 6 | 0.81660 | 11 | 0.66438 | 16 | 0.52251 | 21 | 0.38976 | 26 | 0.29748 |
| 1.5 | 0.95738 | 6.5 | 0.80041 | 11.5 | 0.65022 | 16.5 | 0.50822 | 21.5 | 0.38053 | 26.5 | 0.28825 |
| 2 | 0.94218 | 7 | 0.78422 | 12 | 0.63606 | 17 | 0.49393 | 22 | 0.37130 | 27 | 0.27902 |
| 2.5 | 0.92633 | 7.5 | 0.76897 | 12.5 | 0.62189 | 17.5 | 0.47965 | 22.5 | 0.36208 | 27.5 | 0.26980 |
| 3 | 0.91048 | 8 | 0.75371 | 13 | 0.60773 | 18 | 0.46536 | 23 | 0.35285 | 28 | 0.26057 |
| 3.5 | 0.89473 | 8.5 | 0.73846 | 13.5 | 0.59357 | 18.5 | 0.45107 | 23.5 | 0.34362 | 28.5 | 0.25134 |
| 4 | 0.87898 | 9 | 0.72321 | 14 | 0.57941 | 19 | 0.43679 | 24 | 0.33439 | 29 | 0.24211 |
| 4.5 | 0.86398 | 9.5 | 0.70796 | 14.5 | 0.56524 | 19.5 | 0.42250 | 24.5 | 0.32516 | 29.5 | 0.23288 |
| 5 | 0.84899 | 10 | 0.69271 | 15 | 0.55108 | 20 | 0.40822 | 25 | 0.31594 | 30 | 0.22366 |

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## Forward Swap Rate (FSR)

Using the calculated OIS discount factors and the Libor forwards (calculated from the Libor discount factors), we can calculate the FSR as shown below:

|  |  |
| --- | --- |
| **Tenor** | **FSR** |
| 1 x 1 | 0.032007 |
| 1 x 2 | 0.033259 |
| 1 x 3 | 0.034011 |
| 1 x 5 | 0.035255 |
| 1 x 1 | 0.038428 |
| 5 x 1 | 0.039274 |
| 5 x 2 | 0.040075 |
| 5 x 3 | 0.040072 |
| 5 x 5 | 0.041093 |
| 5 x 10 | 0.043634 |
| 10 x 1 | 0.042190 |
| 10 x 2 | 0.043116 |
| 10 x 3 | 0.044097 |
| 10 x 5 | 0.046249 |
| 10 x 10 | 0.053458 |

# Part II – Model Calibration

## Displaced Diffusion Model

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Sigma** | | | | | |
| **Expiry\Tenor** | **1Y** | **2Y** | **3Y** | **5Y** | **10Y** |
| **1Y** | 0.307542 | 0.345404 | 0.341398 | 0.289518 | 0.259990 |
| **5Y** | 0.302838 | 0.312472 | 0.308905 | 0.272008 | 0.247628 |
| **10Y** | 0.296200 | 0.297524 | 0.294184 | 0.267308 | 0.243554 |
| **Beta** | | | | | |
| **Expiry\Tenor** | **1Y** | **2Y** | **3Y** | **5Y** | **10Y** |
| **1Y** | 0.012154 | 0.015845 | 0.031462 | 0.111442 | 0.295188 |
| **5Y** | 0.052560 | 0.107758 | 0.144572 | 0.249040 | 0.367633 |
| **10Y** | 0.121313 | 0.180841 | 0.146493 | 0.286947 | 0.386948 |

## SABR Model

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| **Alpha** | | | | | |
| Expiry\Tenor | **1Y** | **2Y** | **3Y** | **5Y** | **10Y** |
| 1Y | 0.139070 | 0.184646 | 0.196851 | 0.178052 | 0.171237 |
| 5Y | 0.166509 | 0.199497 | 0.210346 | 0.191011 | 0.177441 |
| 10Y | 0.177551 | 0.195043 | 0.207212 | 0.201519 | 0.180061 |
| **Rho** | | | | | |
| **Expiry\Tenor** | **1Y** | **2Y** | **3Y** | **5Y** | **10Y** |
| **1Y** | -0.63322 | -0.52512 | -0.48284 | -0.41443 | -0.26565 |
| **5Y** | -0.58516 | -0.54687 | -0.54976 | -0.51106 | -0.44079 |
| **10Y** | -0.54586 | -0.54376 | -0.55087 | -0.56245 | -0.50698 |
| **Nu** | | | | | |
| **Expiry\Tenor** | **1Y** | **2Y** | **3Y** | **5Y** | **10Y** |
| **1Y** | 2.049482 | 1.677437 | 1.438138 | 1.064877 | 0.776535 |
| **5Y** | 1.339619 | 1.061909 | 0.936718 | 0.671724 | 0.493849 |
| **10Y** | 1.007291 | 0.924810 | 0.869132 | 0.719875 | 0.579936 |

## Pricing Swaptions

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| ***2 x 10 Payer Swaption*** | | | ***8 x 10 Receiver Swaption*** | | |
| ***K*** | ***Displaced Diffusion*** | ***SABR*** | ***K*** | ***Displaced Diffusion*** | ***SABR*** |
| *1%* | *0.287825* | *0.290670* | *1%* | *0.010445* | *0.019217* |
| *2%* | *0.194357* | *0.199352* | *2%* | *0.024849* | *0.038276* |
| *3%* | *0.112743* | *0.116159* | *3%* | *0.049254* | *0.060689* |
| *4%* | *0.054014* | *0.052858* | *4%* | *0.085163* | *0.089214* |
| *5%* | *0.021025* | *0.021666* | *5%* | *0.132747* | *0.128133* |
| *6%* | *0.006680* | *0.010836* | *6%* | *0.191083* | *0.182277* |
| *7%* | *0.001762* | *0.006650* | *7%* | *0.258592* | *0.252024* |
| *8%* | *0.000394* | *0.004645* | *8%* | *0.333466* | *0.332294* |

Based on the result above, we can observe that as strike goes larger, the price of payer becomes lower, but the price of receiver becomes higher, and both models’ results are very close to each other. However, since SABR does a better job in fitting market implied volatility, the result from SABR model is more accurate than Displaced Diffusion Model (please refer to next page for visualized comparison).

## Model Calibration Plots

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# Part III – CMS Static Replication

## Constant Maturity Swap Valuation

A constant maturity swap (CMS) pays a swap rate rather than a LIBOR rate on its floating leg. CMS products gives us an easy way to gain exposure to fixed-length longer-term interest rates by taking a view on a fixed point on the yield curve. The standard practice in the market is to use the static-replication method to obtain a model-independent convexity correction. By static-replication approach, and choosing the forward swap rate F = Sn,N (0) as our expansion point.

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**IRR-settled option pricer (V-Pay or V-Rec) is given by:**

*whereby Sigma for Black76 model is derived from the SABR model*

From the calibrated SABR parameters in Part II, we can derive the sigma used in the Black76 lognormal model. This enables us to obtain the IRR settled option for both receiver and payer swaption. From there we can get the CMS rate followed by the PV of CMS leg. Obtaining the CMS rate requires the SABR cubic spline interpolated Alpha, Rho & Nu parameters (in Appendix) since these parameters are not linear across time.

**CMS Rate is written as:**



**PV of a CMS leg is the sum of the discounted values of the CMS rates, multiplied by the day count fraction:**

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**Hence:**

* PV of a leg receiving CMS10y semi-annually over the next 5 years is **0.20209**.
* PV of a leg receiving CMS2y quarterly over the next 10 years is **0.38106**.

## **Chart, line chart Description automatically generated**Effects on Convexity Correction

|  |  |  |  |
| --- | --- | --- | --- |
| ***Comparison of CMS Rates & Forward Swap Rates*** | | | |
| **Tenor** | **CMS Rate** | **FSR Rate** | **Difference** |
| 1 x 1 | 0.032120 | 0.032007 | 0.000113 |
| 1 x 2 | 0.033382 | 0.033259 | 0.000122 |
| 1 x 3 | 0.034120 | 0.034011 | 0.000109 |
| 1 x 5 | 0.035326 | 0.035255 | 0.000070 |
| 1 x 10 | 0.038496 | 0.038428 | 0.000068 |
| 5 x 1 | 0.040129 | 0.039274 | 0.000855 |
| 5 x 2 | 0.040756 | 0.040075 | 0.000681 |
| 5 x 3 | 0.040664 | 0.040072 | 0.000591 |
| 5 x 5 | 0.041532 | 0.041093 | 0.000439 |
| 5 x 10 | 0.044050 | 0.043634 | 0.000416 |
| 10 x 1 | 0.043635 | 0.042190 | 0.001446 |
| 10 x 2 | 0.044323 | 0.043116 | 0.001206 |
| 10 x 3 | 0.045225 | 0.044097 | 0.001128 |
| 10 x 5 | 0.047210 | 0.046249 | 0.000961 |
| 10 x 10 | 0.054528 | 0.053458 | 0.001069 |

**Chart, line chart

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CMS convexity correction is the difference between the expected CMS rate and the implied forward swap rate (under the swap measure).

**Chart, line chart

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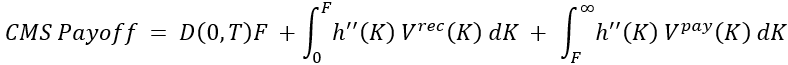
From our result, we see that this is true with CMS rates always being higher than FSR rates. The difference between CMS rate and FSR rate is a function of expiry, the longer the maturity the greater the effects of convexity correction.

# Part IV – Decompounded Options Static Replication

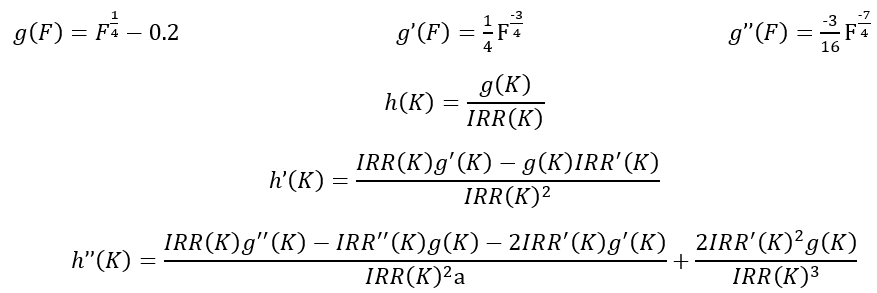
## Decompounded Option Valuation

Payoff:(Decompounded Option)

The Static Replication equation can be represented using the IRR payer and receiver swaption. The formula is stated below:



Below is the equation necessary to complete the derivation:



By substituting the 5 x 1 FSR, 5Y OIS DF and SABR parameters, the PV is **0.1773379**.

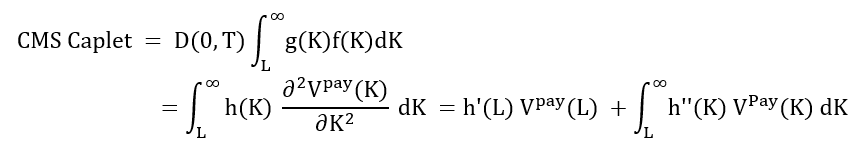
## CMS Caplet Valuation

Payoff:  (CMS Caplet)

To have a positive payoff there is a condition that needs to be satisfied:



The formula for CMS caplet with strike of 0.0016 will be:



By substituting the FSR and SABR parameters, the PV is **2.6351730**.

# Appendix:

## Calibrated SABR Parameters with Cubic Spline Interpolation for CMS Rate

**Alpha**

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**Rho**

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**Nu**

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